A

Synopsis

On

OF THE PROPOSED THESIS ENTITLED

‘THE STUDY OF ZERO DIVISOR GRAPH OF RINGS AND MODULES’

Submitted to

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1.Abstract

The main object of the subject is to study the interplay of ring-theoretic properties of R with graph-theoretic properties of $\Gamma(R)$. This study helps to illuminate the structure of $Z(R)$.

In this research work we will include some basic definition from graph theory as needed. Also we will use some references for graph theory for commutative ring theory.

2.Introduction

The zero divisor graph is an undirected graph representing the zero divisors of a commutative ring. The zero-divisor graphs of commutative rings is the study which gives relation between zero-divisors and graph theory.

Let R be a commutative ring with unity and let $Z(R)$ be its set of zero divisors. We associate a graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R)-\{0\}$ the set of nonzero zero-divisors of R, and for disinct $x, y \in Z(R)^*$, the vertices x *and* y are adjacent *iff* $x \neq y$ *and* $x.y = 0$. Thus $\Gamma(R)$ is the empty graph if and only if R is an integral domain. The zero divisor grph gives the interplay of ring-theoretic properties of R with graph-theoretic properties *of* $\Gamma(R)$.

The zero-divisor graph of a ring R is used to establish a bound on the cardinality of R. The concept of a zero-divisor graph was introduced by I. Beck in *1988*. All the elements of R be vertices of the graph and they were mainly interesting in colorings. The first simplification of Becks zero-divisor graph was introduced by D.F.Anderson and P.S.Livingston. Their Aim is to find better pictures of the zero divisor structure of the ring. Further the S.P.Redmond introduced the ideal-based zero divisor graph $\Gamma(R)$. The Compressed zero divisor graph using different notations was first defined by S.B.Mula. The Semi group zero-divisor graph was given by F.R. De Meyer, T.Mckenzie.

Usually, the ring of integers and integers modulo n will be denoted by $\mathbb{Z}$ and $\mathbb{Z}_n$ respectively & F_q will be the finite field with q elements we show that $\Gamma(R)$ is always connected and has small diameter & girth, here we determine which complete graphs and star graphs may be realized as $\Gamma(R)$.

3. Review of Literature

In the zero divisor graph, so much literature has been written from very different points of view, here we collect an over view of the material.

I . Beck was given the concept of zero divisor graph. Next the zero divisor graph studied by Anderson and Nasser, they all interested in colorings. Let *R* be a ring and the elements of *R* be vertices of the graph. Further the S.P.Redmond introduced the ideal-based zero divisor graph $\Gamma(R)$. The compressed zero divisor graph using different notations was first defined by S.B.Mula. The semi group zero-divisor graph was given by F.R.De Meyer, T.Mckenzie.

Let *R* be a commutative ring with identity, the zero-divisor graph $\Gamma(R)$,is the graph with edges between distinct vertices x and y whenever $xy=0$, and vertices the nonzero zero divisors of *R*. The centre union *0* is shown to be a union of annihilator ideals if *R* is Artinian.

The annihilator ideal is the diameter of $\Gamma(R)$ can be determined when the centre is identified. The Anderson given the definition, let *R* be a commutative ring with identity, *Z(R)* its set of zero divisors and *Nil(R)* its ideal of Nilpotent elements. The Nilpotent element is defined as,

The zero-divisor graph of *R* is $\Gamma(R)= Z(R)-\{0\}$ with distinct vertices *x and y* adjacent if and only if $xy = 0$

4. Importance of Research

This subject appealing since techniques can vary from simple computations to quite sophisticated ring theory and in many cases. All the rings or graphs satisfying a certain property can be explicitly listed.

We know that Anderson and Livingston completely characterized the star graphs of the form $\Gamma(R)$ where R is finite, It is proved that the star graphs *G* that occur as $\Gamma(R)$ are precisely those. Such that $|G|$ is a prime power. This things leads to the question of whether or not a star graph of any size could be realized as $\Gamma_E(R)$. Also the thesis is about the preliminaries and notations related to zero divisor graph of a commutative ring *R* , we will derive the chromatic number of a zero divisor graph $\Gamma[\mathbb{Z}_n]$, we calculate the clique number of $\Gamma[\mathbb{Z}_n]$

In terms of the classification of these zero divisor graphs, we will investigate cut vertices, girth and edge domination when *R* is Noetherian . Any how in particular we show that a cut vertex $(\Gamma_E(R))\leq\infty$ Correspond to an associated prime, and the girth of the graph.

Continuing with the previous research on zero divisor graphs, we also consider graph homomorphism & colorings respectively, we are able to address a version of becks conjecture regarding chromatic number for the graph $\Gamma_E(R)$ by constructing a ring R such that $\omega(\Gamma_E(R))=3$, but $\chi(\Gamma_E(R))=4$.

The development of algorithms to handle graphs is major interest in computer science. By knowing the applications and use of graph theory in every field and science, we decided to study 'Graph theory' to develop new results. Zero divisor graph is nothing but the relation between ring theory and graph theory. So by using this we can able to understand most complicated structure of ring (which are not easy to visualize) with the help of graph. In this way the properties of ring can be easily manipulated with the properties of graph.

After this we will able to relate the properties of graph such as girth, chromatic no., connectivity, diameter, sub-graph, etc with the ring theoretic properties such as ideal, sub-ring, etc.

5. Methodology to be adopted

The main object is to study the interplay of ring-theoretic properties of R with graph-theoretic properties $\Gamma(R)$. An algorithm is presented to find all commutative reduced rings with 1 which give rise to a zero-divisor graph on n vertices for any $n \geq 1$ the zero-divisor graph of a finite commutative ring is used to find bounds on the size of that ring.

We will study, the Zero divisor graphs of matrices over commutative rings & zero divisor graphs of the ring of Lipschitz integers modulo n. A zero divisor graph is an undirected graph representing the zero divisors of a commutative ring. It has elements of the ring as its vertices and pairs of elements whose product is zero as its edges.

In the whole research we will use the methodology as, some Definitions & Notations like, Ring, Commutative ring, Undirected Graph, Cycle, Zero divisor Graph, Annihilator, Noetherian, Ring etc. In zero divisor graph we will gone through Simple graph, Complete graph, Bipartite graph. The Zero-Divisor graph of a Commutative ring and also Mulay's Zero Divisor graph.

6. Objective of research / proposed hypothesis

The present objective of the work is to study and determine the following parameters.

- Zero-divisor graphs.
- To make the relation between the properties of ring and properties of graphs.
- Zero-Divisor Graphs of matrices over commutative Rings.
- To study the basic properties such as connectivity, diameter and girth of the zero divisor graphs.
- Zero divisor graph of the Ring of Lipschitz integers modulo *n*.
- To established ideal- based zero divisor graphs.
- Energy of the zero-divisor graphs.
- Wiener index of the zero divisor graph.
- To calculate diameter and girths of zero divisor graphs of commutative ring and compare the result with the graph of non commutative ring.
- Chromatic number of the zero divisor graph.
- To evaluate degree of vertex of zero divisor graph and relate its properties with ring and the ring theoretic properties.
- To study different commutative and non commutative rings and their relevant undirected and directed zero divisor graphs respectively.
- Clique number of the zero divisor graph.
- To find different homomorphism of ring *R* and find their respective zero divisor graph and determine the relation between graph of *R* and corresponding sub graphs of relevant zero divisor graphs.

7. Significance of the study

The research work introduces several new kinds of zero-divisor type graphs.

Though we investigate a great deal here the adherence of many properties proven, for graphs.

Graph theory is used to study molecules in Chemistry and Physics, three-dimensional structure of complicated simulated atomic structure can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms. This approach is especially used in Computer processing of molecular structures, ranging from chemical editors to database searching. Similarly in computational neuroscience graphs can be used to represent functional connections.

8. Plan of research work

First Year

Some basic materials required for the research and field work. This includes collection of journals, Books & some research papers related to the topic. To study collected materials .

Second Year

The collected material will be processed, analyzed on direction of the research topic. The created outcome will be presented in national & international gathering such as seminars, workshops, conferences etc. To study processed materials through direction of the research topic and attempt will be created.

Third Year

The outcome result will be presented. Also will be publishe the outcome results in national, international journals during this year. Drafting, Writing & Submission of final research work done report to Swami Ramanand Teerth Marathwada University, Nanded.

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